Math 181 (Study Guide For The Final Exam)

This study guide consists of only a few sample of questions, in order to prepare fully you should know all the sections that we covered for this course and the related examples we did in the class as well as previous study guides.

1. \[ \lim_{\theta \to 0} 4 \frac{\sin(\sin \theta)}{\sec \theta} \]
   \[ \text{ANS: } 0 \] \hspace{1cm} (3.3)

2. Calculate \( y' \). \[ y = 3 \sqrt{x} \cos \sqrt{x} \]
   \[ \text{ANS: } y' = - \frac{3}{2} \left( \frac{\sqrt{x} \sin \sqrt{x} - \cos \sqrt{x}}{\sqrt{x}} \right) \]

3. Differentiate. \[ y = \frac{\sin x}{3 + \cos x} \]
   \[ \text{ANS: } \frac{dy}{dx} = \frac{3\cos(x) + 1}{(3 + \cos(x))^2} \] \hspace{1cm} (3.4)

4. The speed of a runner increased steadily during the first three seconds of a race. Her speed at half-second intervals is given in the table. Find a lower estimate for the distance that she traveled during these three seconds.

<table>
<thead>
<tr>
<th>( t \text{(s)} )</th>
<th>0</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{(ft/s)} )</td>
<td>0</td>
<td>2.8</td>
<td>3.5</td>
<td>6.9</td>
<td>8.2</td>
<td>12.2</td>
<td>16.3</td>
</tr>
</tbody>
</table>

\[ \text{ANS: } 16.8 \] \hspace{1cm} (5.1)

5. Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.
   \[ h(x) = \int_1^x 2 \ln t \, dt \]
   \[ \text{ANS: } 2xe^x \] \hspace{1cm} (5.3)

6. Find the derivative of the function. \[ g(x) = \int_4^x 4 \sqrt{1 + t^2} \, dt \]
   \[ \text{ANS: } 8x \sqrt{1 + x^4} \] \hspace{1cm} (5.3)

7. Evaluate the indefinite integral.
   \[ \int \sqrt{4x} \sin \left( \frac{5 + x^{3/2}}{2} \right) \, dx \]
   \[ \text{ANS: } - \frac{4}{3} \cos \left( \frac{5 + (\sqrt{x})^{3}}{3} \right) \] \hspace{1cm} (5.5)
8. Evaluate the integral by making the given substitution. \( \int \cos(5x)\,dx \)

\[ \frac{1}{5} \sin 5x + C \]  
ANS: \( \frac{1}{5} \sin 5x + C \) (5.5)

9. Evaluate the integral.

\[ \int_0^1 7x^6 \cos\left(\pi x^7\right)\,dx \]

ANS: \( \sin(1) \) (5.5)

10. Evaluate the indefinite integral.

\[ \int 4x\left(x^2 + 3\right)^4 \,dx \]

ANS: \( \frac{2}{5} \left(x^2 + 3\right)^5 + C \) (5.5)

11. Evaluate the integral.

\[ \int_0^1 x^2\left(5 + 2x^3\right)^2 \,dx \]

ANS: \( \frac{109}{9} \) (5.5)

12. The position of a car is given by the values in the following table.

<table>
<thead>
<tr>
<th>( t ) (seconds)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s ) (meters)</td>
<td>0</td>
<td>23.7</td>
<td>27</td>
<td>58.3</td>
<td>90.5</td>
</tr>
</tbody>
</table>

Find the average velocity for the time period beginning when \( t = 2 \) and lasting 2 seconds.

ANS: \( 31.75 \) (2.1)

13. Sketch the graph of the function \( f \) and evaluate \( \lim_{x \to -3^+} f(x) \).

\[ f(x) = \begin{cases} 
  x + 5, & \text{if } x \leq -3 \\
  -2x - 1, & \text{if } x > -3 
\end{cases} \]

ANS: \( 5 \) (2.2)
14. Use the graph of \( f(x) = \frac{x^2 - x - 2}{x - 2} \) to guess at the limit \( \lim_{x \to 2} \frac{x^2 - x - 2}{x - 2} \), if it exists.

ANS: \( 3 \) \hspace{1cm} (2.2)

15. Find the \( \lim_{x \to 3} \frac{x^2 - x - 6}{x - 3} \), if it exists.

ANS: \( 5 \) \hspace{1cm} (2.3)

16. Find the \( \lim_{x \to 0} \frac{\sqrt{x + 14} - \sqrt{14}}{x} \), if it exists.

ANS: \( \frac{\sqrt{14}}{28} \) \hspace{1cm} (2.3)

17. Find the \( \lim_{t \to 0} \frac{t^2 + 3}{t^3 + t^2 - 7} \)

ANS: \( 0 \) \hspace{1cm} (2.5)

18. How would you define \( f(3) \) in order to make \( f \) continuous at 3? \( f(t) = \frac{t^2 + t - 12}{t - 3} \)

ANS: \( f(3) = 7 \) \hspace{1cm} (2.5)

19. For what value of the constant \( c \) is the function \( f \) continuous on \((-\infty, \infty)\)?

\[
f(x) = \begin{cases} 
  cx + 7 & \text{for } x \leq 2 \\
  cx^2 - 5 & \text{for } x > 2
\end{cases}
\]

ANS: \( c = 6 \) \hspace{1cm} (2.5)
20. If \( g(x) = \sqrt{4 - 3x} \), find the domain of \( g'(x) \).  
ANS: \((-\infty, \frac{4}{3})\)  
(2.8)

21. Find the second derivative of the function. 
\( f(x) = 5e^x \cos x \)  
ANS: 
\[-10e^x \sin x\]  
(3.1)

22. Find the second derivative of the function. 
\( f(x) = x \left(2x^2 - 1\right)^6 \)  
ANS: 
\[24x \left(2x^2 - 1\right)^4 \left(26x^2 - 3\right)\]  
(3.1)

23. Find an equation of the line tangent to the graph of \( y = \frac{e^{9x}}{x^2 + 1} \) at the point where \( x = 0 \).  
ANS: \( y = -9x + 1 \)  
(3.1)

24. Find the point(s) on the graph of \( f \) where the tangent line is horizontal. 
\( f(x) = x^8 e^{-x} \)  
ANS: \( \left(0, 0\right), \left(e^8, \frac{8^8}{e^8}\right) \)  
(3.4)

25. Find \( \frac{dy}{dx} \) by implicit differentiation. 
\( e^y - x^8 + y^8 = 8 \)  
ANS: 
\[\frac{8x^7 - ye^y}{xe^y + 8y^7}\]  
(3.5)

26. Find \( \frac{d^2y}{dx^2} \) in terms of \( x \) and \( y \). 
\( x^6 - y^6 = -1 \)  
ANS: 
\[\frac{5x^4}{y^5} - \frac{5x^{10}}{y^{11}}\]  
(3.5)

27. Calculate \( y' \). 
\( y = 4 \ln \left(x^2 e^{10}\right) \)  
ANS: 
\[y' = 4 \left(\frac{2}{x} + 1\right)\]  
(3.6)
28. Differentiate the function. \( h(t) = \frac{\ln 6t}{\ln 12t} \)  
ANS: \( \frac{\ln 2}{t(\ln 12t)^2} \) (3.6)

29. Find the critical number(s), if any of the function \( f(t) = 2t^\frac{3}{4} - 4t^\frac{1}{4} \).  
ANS: \( \frac{4}{9}, 0 \) (4.1)

30. Find the critical number(s), if any, of the function \( f(x) = e^{x^3 - x} \).  
ANS: 1/2 (4.1)

31. Find the absolute maximum and absolute minimum values, if any, of the function \( f(x) = x - \sqrt{x} \) on [0, 25]  
ANS:  
Abs. Max. \( f(25) = 20 \) (4.1)  
Abs. Min. \( f(1/4) = -1/4 \)

32. Determine where the graph of the function \( f(x) = 9x - \sqrt{4-x^2} \) is concave upward.  
ANS: CU on (-2, 2), (4.3)

33. Use the Second Derivative Test to find the relative extrema, if any, of the function \( f(x) = 2x^3 - 3x^2 - 36x - 5 \).  
\( \text{ANS: } \text{Rel. Max. } f(-2) = -86, \text{ Rel. Min. } f(3) = -86 \) (4.3)

34. How many points of inflection are on the graph of the function?  
\( f(x) = 18x^3 + 5x^2 - 12x - 20 \)  
ANS: 1 (4.3)

35. Given \( f(x) = 2x^3 - 3x^2 - 36x + 6 \).  
(a) Find the intervals on which \( f \) is increasing or decreasing.  
(b) Find the relative maxima and relative minima of \( f \).  
\( \text{ANS: (a) Increasing on } (-\infty, -2) \text{ and } (3, \infty), \text{ decreasing on } (-2, 3) \)  
(b) Rel. Max. \( f(-2) = 50 \), \( \text{Rel. Min. } f(3) = -75 \) (4.3)
36. Find \(\lim_{u \to 2^+} \frac{-5u^2}{u-2}\) \hspace{1cm} ANS: \(-\infty\) (4.4)

37. Evaluate \(\lim_{x \to 0^+} \frac{2e^x + x - 2}{2 - \sqrt{4 - x^2}}\) \hspace{1cm} ANS: \(\infty\) (4.4)

38. Evaluate \(\lim_{x \to 0} \frac{(7 \sin x)^2}{1 - \sec x}\) \hspace{1cm} ANS: \(-98\) (4.4)

39. Evaluate the \(\lim_{x \to \infty} \left( x - \sqrt{x^2 + 2} \right)\) \hspace{1cm} ANS: 0 (4.4)

40. Find \(f\), if \(f''(x) = 18x + 24x^2\) \hspace{1cm} ANS: \(f(x) = 3x^3 + 2x^4 + Cx + D\) (4.9)

41. Find \(f\), \(f'(t) = 2t - 4 \sin t\), \(f(0) = 5\) \hspace{1cm} ANS: \(f(t) = t^2 + 4 \cos t + 1\) (4.9)

42. Evaluate the Riemann sum for \(f(r) = 36 - r^2\), \(0 \leq r \leq 2\) with four subintervals, taking the sample points to be right endpoints. \hspace{1cm} ANS: 68.25 (5.2)

43. Use the Midpoint Rule with \(n = 10\) to approximate the integral. \(\int_1^2 \sqrt{4 + t^2} \, dt\) \hspace{1cm} ANS: 2.510716 (5.2)

44. Find the general indefinite integral. \(\int \frac{\sin 120t}{\sin 60t} \, dt\) \hspace{1cm} ANS: \(\frac{\sin 60t}{30} + C\) (5.4)

45. Evaluate the integral. \(\int_1^t \frac{x^2 + 6}{\sqrt{x}} \, dx\) \hspace{1cm} ANS: 24.4 (5.4)
46. Evaluate the integral if it exists.  
\[ \int \left( \frac{5 - x}{x} \right)^2 dx \]
ANS:
\[ x - 10 \ln x - \frac{25}{x} + C \]  
(5.4)

47. Sketch the graph of the function \( f(x) = \frac{x^2 - 16}{x^2 - 9} \) using the curve-sketching guidelines.
You should know how to find all intercepts and Asymptotes as well as increasing/decreasing, all concavities intervals, Inflection point(s) if any for graphing.
ANS:

48. The owner of a ranch has 4000 yd of fencing with which to enclose a rectangular piece of grazing land situated along a straight portion of a river. If fencing is not required along the river, what are the dimensions of the largest area he can enclose? What is the area?

ANS: Dimensions: 2000 yd \( \times \) 1000 yd, Maximum Area: 2,000,000 \( yd^2 \)  
(4.7)

49. Find the position function of a particle moving along a coordinate line that satisfies the given condition.
\[ v(t) = 3t^2 - 4t + 4, \quad s(1) = -1 \]
ANS:
\[ t^3 - 2t^2 + 4t - 4 \]  
(4.9)